

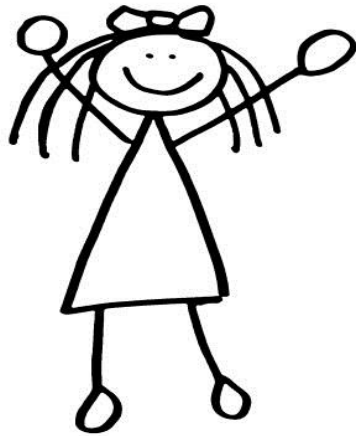
Eurocrypt 2018

# Overdrive: Making SPDZ Great Again

Marcel Keller, Valerio Pastro, and **Dragos Rotaru**

University of Bristol, Yale University, KU Leuven

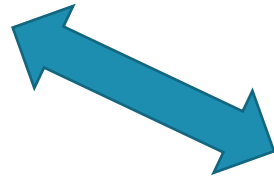
# What's all the fuss about?



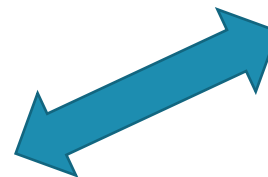
a



c



b



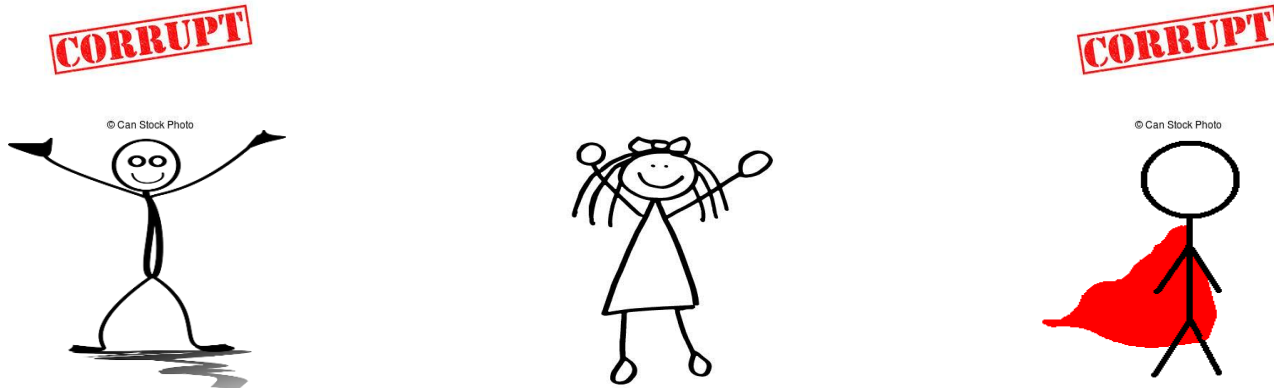
**Goal: Compute  $F(a, b, c)$**

# Security Model



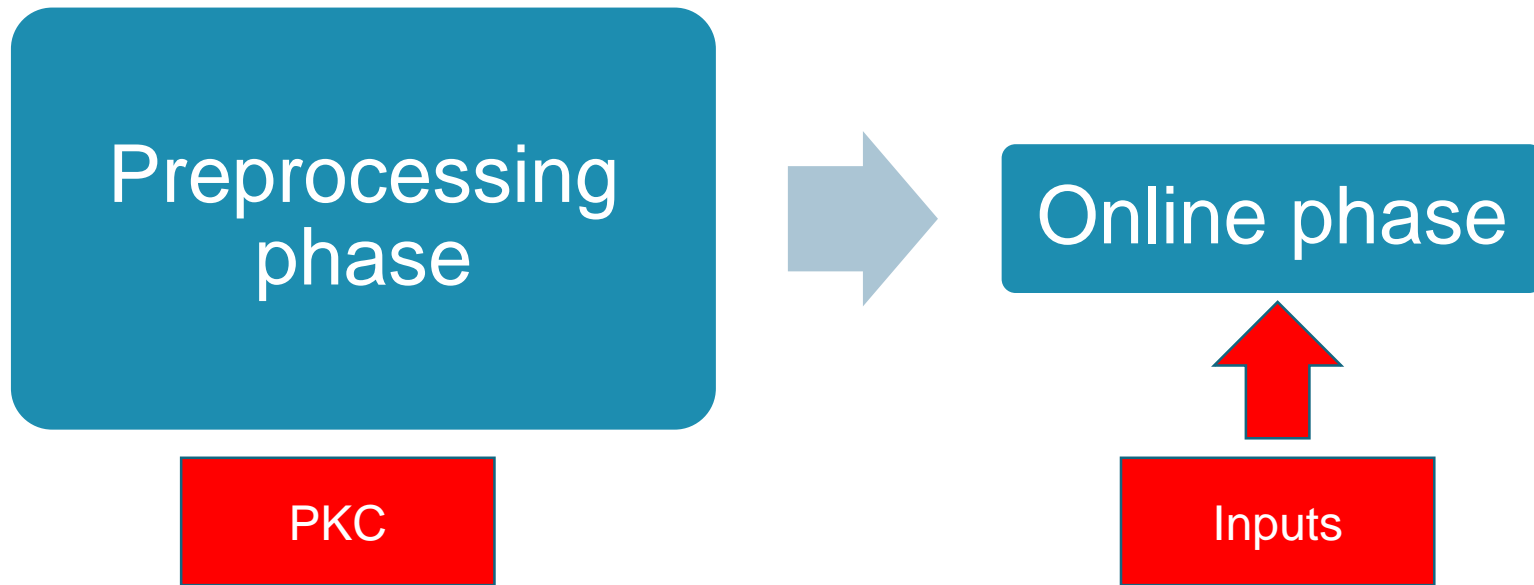
- Many parties (up to  $N$ )
- **Malicious** adversary
- Dishonest majority of corrupted parties

# Security Model



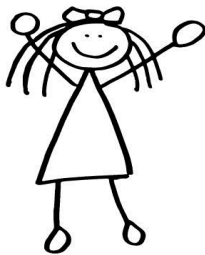
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# Malicious MPC protocols



SPDZ, TinyOT, BDOZa, MASCOT

# Secret share then authenticate



$$\alpha_1 + \alpha_2 + \alpha_3 = \alpha$$

---

$$x_1 + x_2 + x_3 = x$$

---

$$\gamma(x)_1 + \gamma(x)_2 + \gamma(x)_3 = \alpha x$$

# Secret share then authenticate

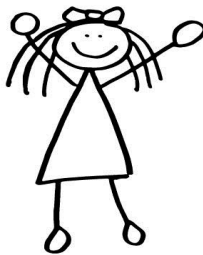


$$\alpha_1 + \alpha_2 + \alpha_3 = \alpha$$

$$(x + y)_1 + (x + y)_2 + (x + y)_3 = x + y$$

$$\gamma(x + y)_1 + \gamma(x + y)_2 + \gamma(x + y)_3 = \alpha(x + y)$$

# Secret share then authenticate



But we want to multiply!



# Let's do it – what do we need?



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# What we have done



# Fastest triple generation!

# How to multiply shared inputs with triples (Beaver's Trick)

$$\begin{aligned}x \cdot y &= (x + a - a) \cdot (y + b - b) \\ &= (x + a) \cdot (y + b) - (y + b) \cdot a - (x + a) \cdot b + a \cdot b\end{aligned}$$

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Masked and opened      Random secret triple

# Revisit, improve, revisit...

BDOZa  
(BDOZ'11)

Semi-homomorphic  
encryption

SPDZ-1  
(DPSZ'12)

Depth-1 SHE  
(NTL), ZK Proof

SPDZ-2  
(DKL+'13)

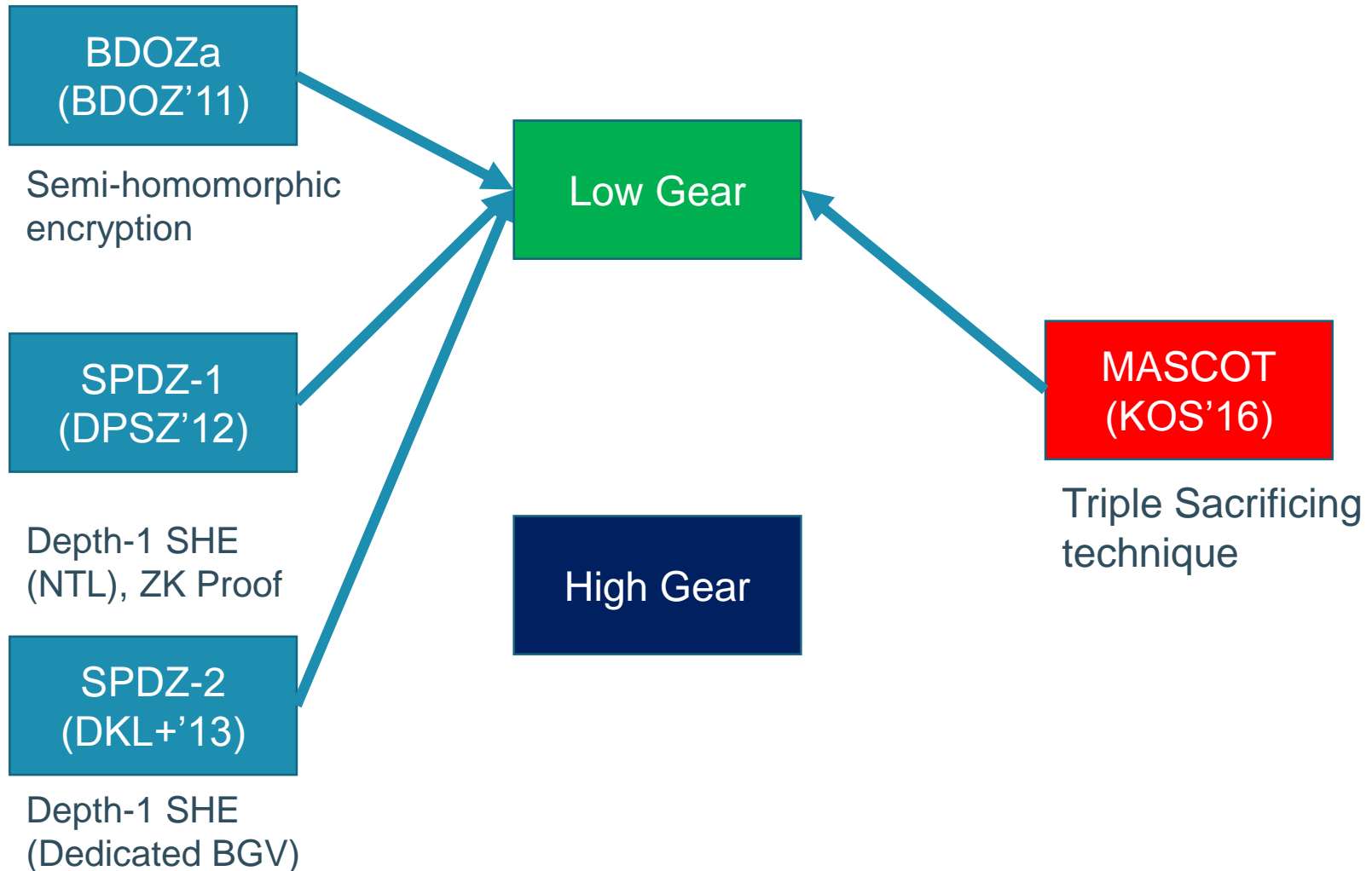
Depth-1 SHE  
(Dedicated BGV)

MASCOT  
(KOS'16)

Triple Sacrificing  
technique



# Revisit, improve, revisit...



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Low Gear

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# LAN Timings

	Triples/s	Security	BGV impl.	$\log_2( \mathbb{F}_p )$
SPDZ-1 [DKL <sup>+</sup> 12]	79	40-bit active	NTL	64
SPDZ-2 [DKL <sup>+</sup> 13]	36	40-bit active	specific	64
SPDZ-1 (ours)	12,000	40-bit active	specific	64
MASCOT [KOS16]	5,100	64-bit active	$\perp$	128
Low Gear (Section 3)	30,000	64-bit active	specific	128

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High Gear

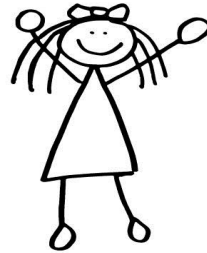
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Triple Sacrificing  
technique

# SPDZ-1 recap



Enc(a[1])  
Enc(b[1])

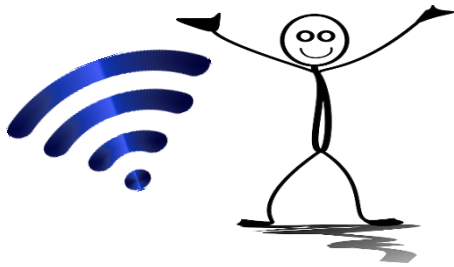


Enc(a[2])  
Enc(b[2])

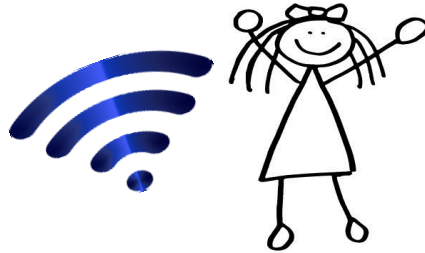


Enc(a[3])  
Enc(b[3])

# SPDZ-1 recap



Enc(a[1])  
Enc(b[1])



Enc(a[2])  
Enc(b[2])



Enc(a[3])  
Enc(b[3])

# SPDZ-1 recap



Enc(a[1])  
Enc(b[1])



Enc(a[2])  
Enc(b[2])

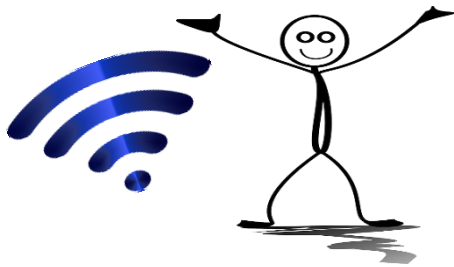


Enc(a[3])  
Enc(b[3])

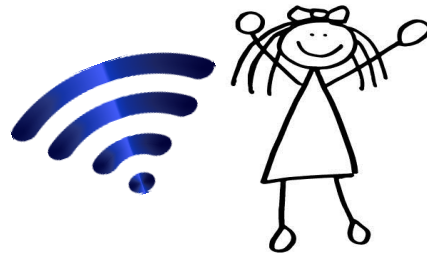
$$C = (\sum_i \text{Enc}(a_i)) \boxtimes (\sum_i \text{Enc}(b_i))$$



# SPDZ-1 recap



Enc(a[1])  
Enc(b[1])



Enc(a[2])  
Enc(b[2])



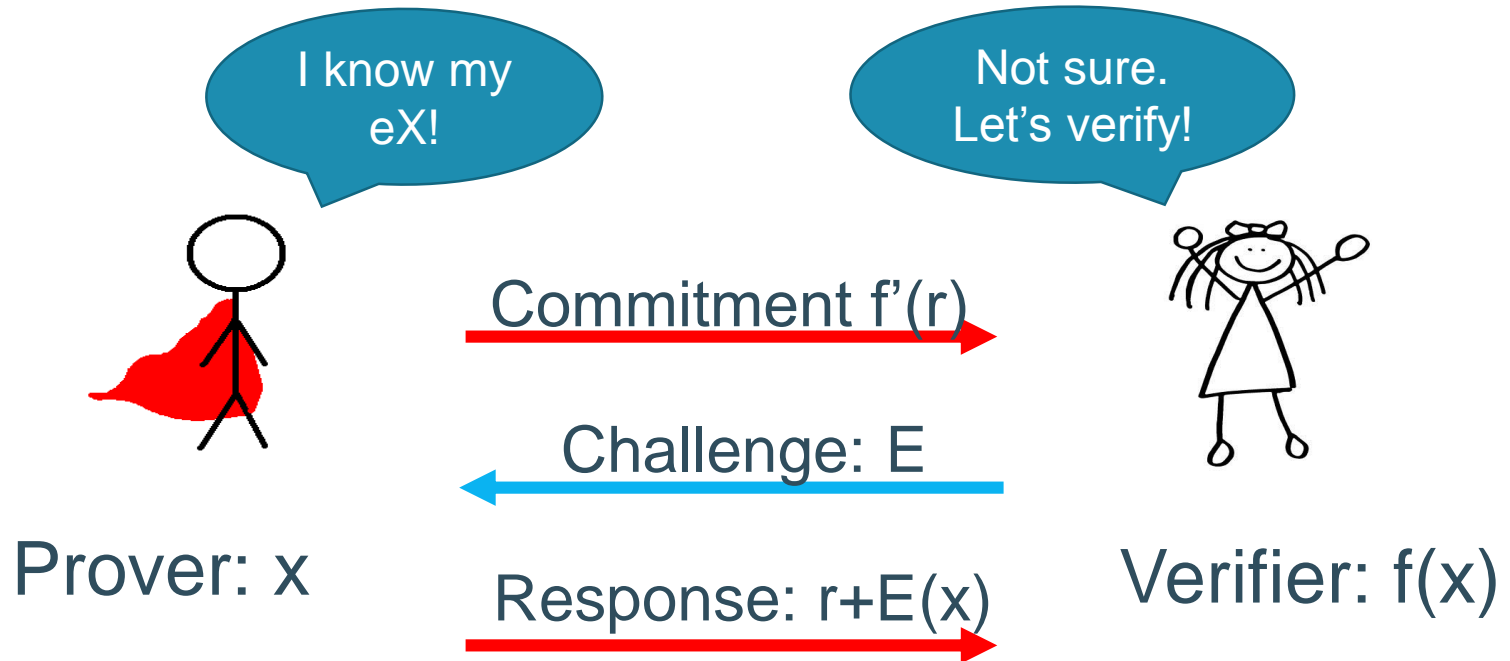
Enc(a[3])  
Enc(b[3])

$$C = (\sum_i \text{Enc}(a_i)) \boxplus (\sum_i \text{Enc}(b_i))$$

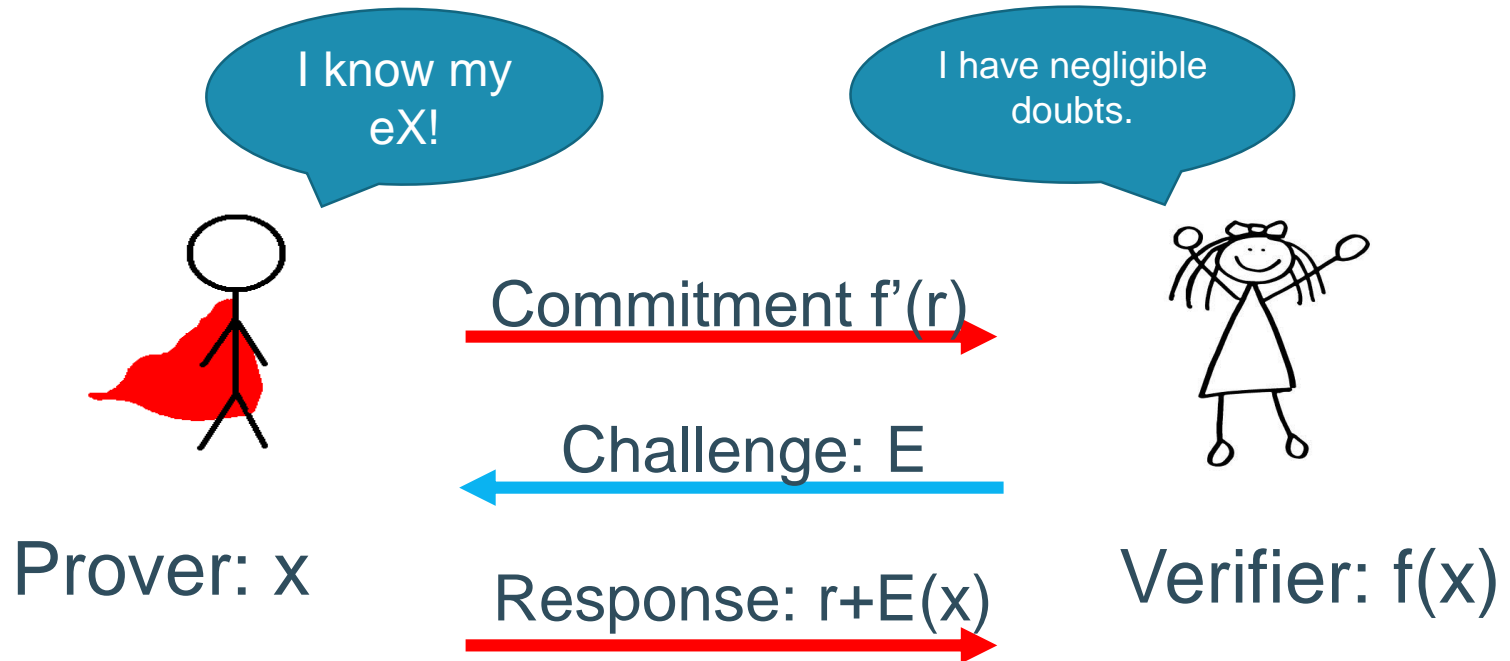
$$C[1] \quad + \quad C[2] \quad + \quad C[3] \quad = \quad C$$

- Parties may lie about their plaintext - incorrect decryption, reveal info about secret keys.
- Need to add ZK proofs for bounding the plaintext

# How to 0-knowledge



# How to 0-knowledge



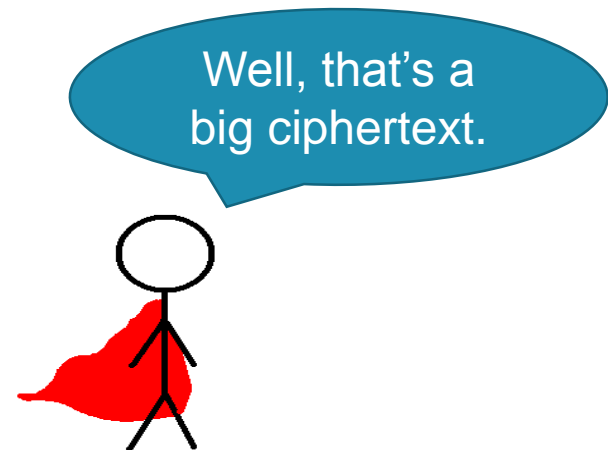
- ?
- $f'(r+E(x)) = f'(r)+E(f(x))$
  - $r+E(x)$  is bounded
  - $r \gg x$ ,  $r/x$  is called slack

# To slack or not to slack

- ZKPoPk: to prove that  $x < B$  we need an encryption scheme which supports plaintexts  $< B * \text{slack}$

Slack is:

- $\sim 2^{50}$  for 40-bit security
- $\sim 2^{100}$  for 128-bit security




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Well, that's a big ciphertext.

- Improve the ZK slack analysis.
- With depth-1 BGV the slack becomes tiny tiny because of the modulus switching.

# Some ciphertexts need no slack

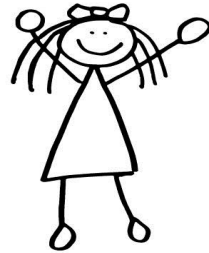
SPDZ			sec	$\log( \mathbb{F}_p )$
1 [DPSZ12]	1 [CDXY17]	2 [DKL <sup>+</sup> 13]		
330	330	332	40	64
572	572	N/A	64	128

**Table 1.** Ciphertext **modulus** bit length ( $\log(q)$ ) for two parties.

# High Gear: SPDZ-1 with global proof



$V(P(\text{Alice}))$   
 $V(P(\text{Bob}))$



$V(P(\text{Bob}))$   
 $V(P(\text{Charlie}))$



$V(P(\text{Alice}))$   
 $V(P(\text{Charlie}))$

---

$V(P(\text{Alice}) + P(\text{Bob}))$

$V(P(\text{Bob}) + P(\text{Charlie}))$

$V(P(\text{Alice}) + P(\text{Charlie}))$

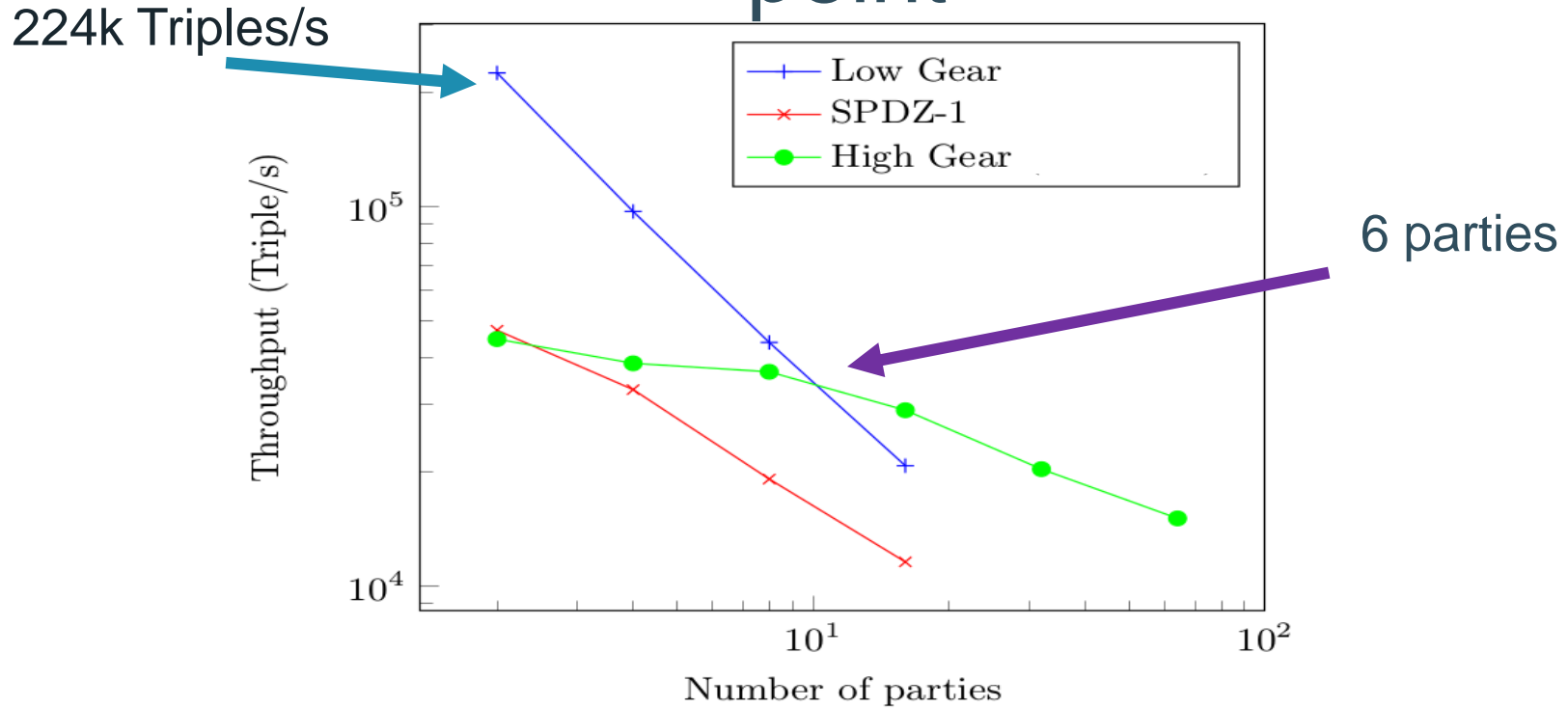


# High Gear: SPDZ-1 with global proof

	Triples/s	Security	$\log_2( \mathbb{F}_p )$
SPDZ-1 (ours)	6,400	64-bit active	128
High Gear (Section 4)	5,600	64-bit active	128



# Low Gear vs High Gear, the tipping point



**Fig. 13.** Triple generation for a 128 bit prime field with 64 bit statistical security on AWS r4.16xlarge instances.

64 CPUs, 488Gb RAM, 25Gb Network

# 100 party Vickrey Auction

AWS instance	Time	Cost per party
t2.nano	9.0 seconds	\$0.000017
c4.8xlarge	1.4 seconds	\$0.000741

**Table 6.** Online phase of Vickrey auction with 100 parties, each inputting one bid.

	Time	Cost per party
MASCOT [ <a href="#">KOS16</a> ]	1,300 seconds	\$0.190
High Gear (Section 4)	98 seconds	\$0.014

**Table 7.** Offline phase of Vickrey auction with 100 parties, each inputting one bid.

AWS m3.2xlarge  
8 CPUs, 30Gb RAM, 10Gb Network

# Code lives on the internetz

<https://github.com/bristolcrypto/SPDZ-2>

Open problem alert:



- In the Low Gear protocol we assumed semi-homomorphic BGV is a linear only encryption scheme.
- Can you create ciphertexts which decrypt to non-linear plaintexts without the KS info? Known as linear target malleability [BCI+13] or linear only encryption [BISW17].

# Thank you!

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- Questions?

# Tiny advert: SCALE at TPMPC

- SCALE (Secure Computation Algorithms from Leuven)
- We do a better analysis of the ZK proofs involved.
- Pre-processing phase coupled with the online phase.
- Compiler is documented, people can read how to use it.
- Others bells and whistles.

