MPC-Friendly Symmetric Key Primitives

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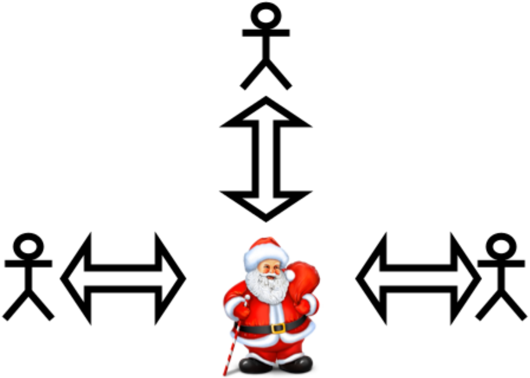
What is Multiparty Computation?



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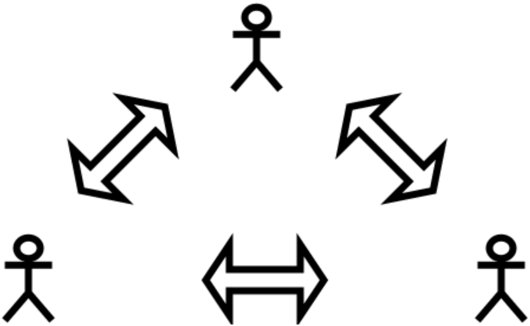
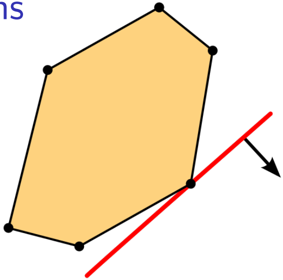
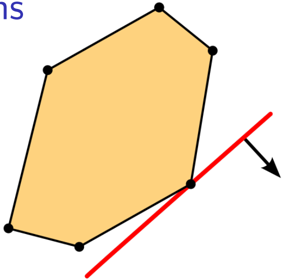


Interesting problems



Linear Programming

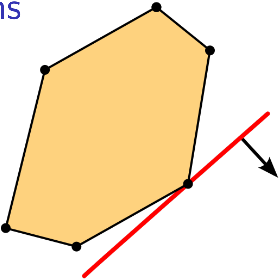
Interesting problems



Linear Programming

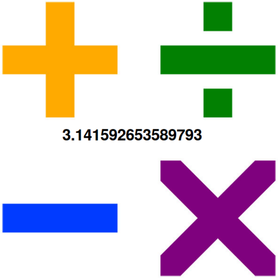
Integer Comparison

Interesting problems



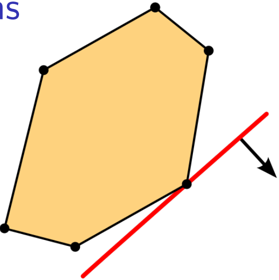
Linear Programming

Integer Comparison



Fixed Point Arithmetic

Interesting problems



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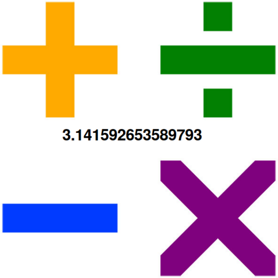
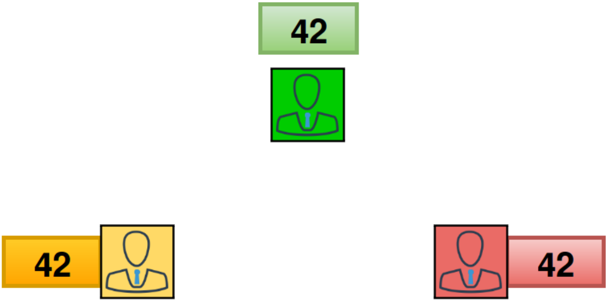


Fixed Point Arithmetic

Interesting problems

Easy to implement via arithmetic circuits mod p

There is a problem.



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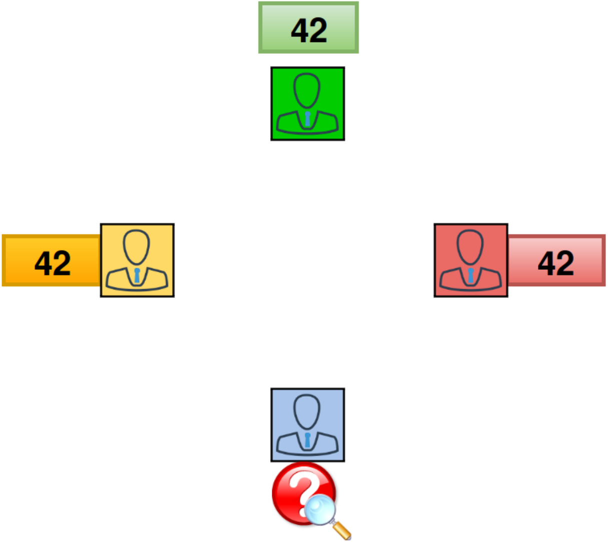

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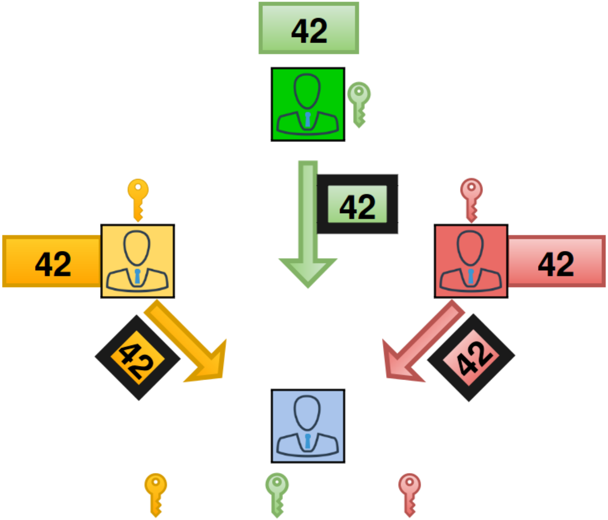

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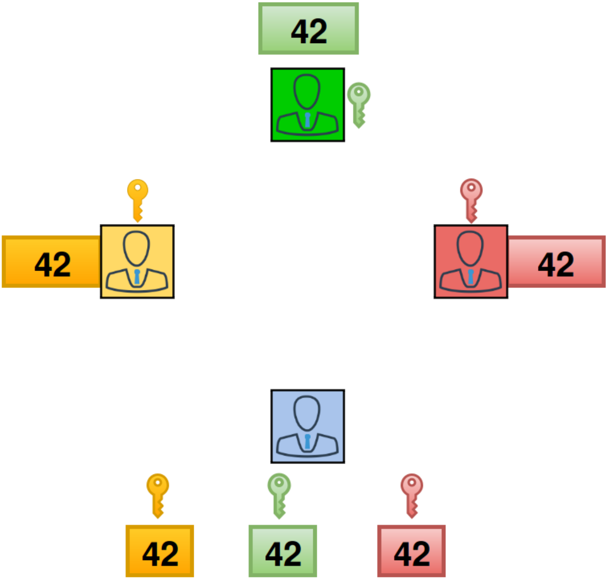

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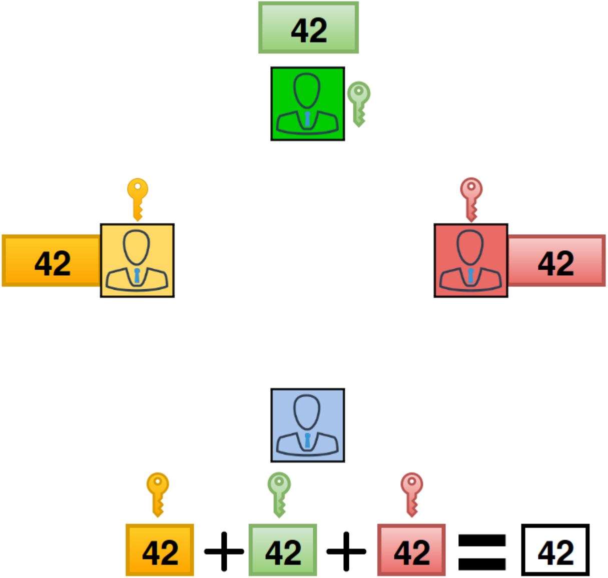

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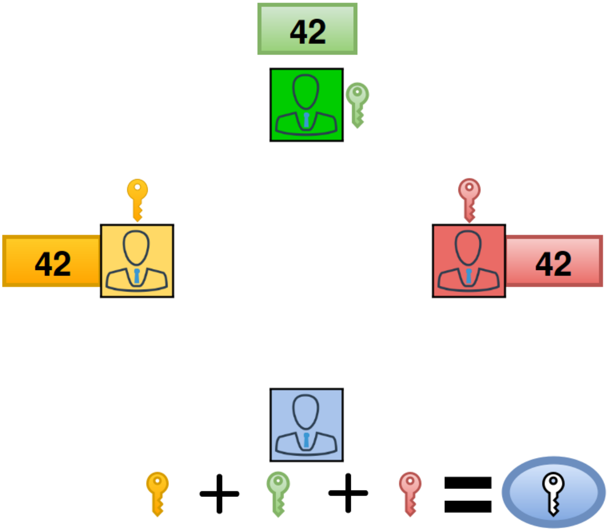

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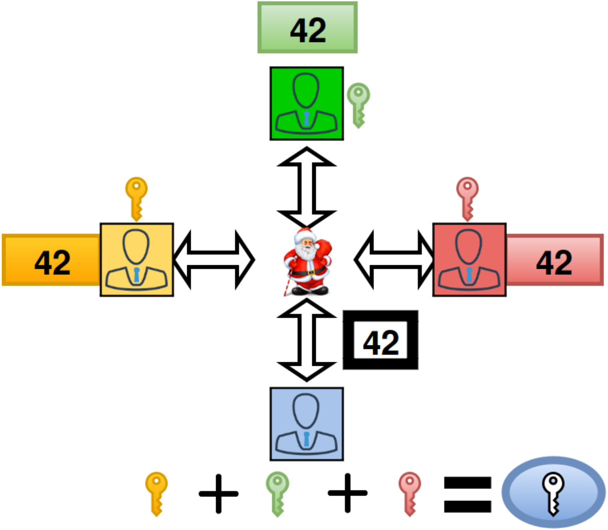

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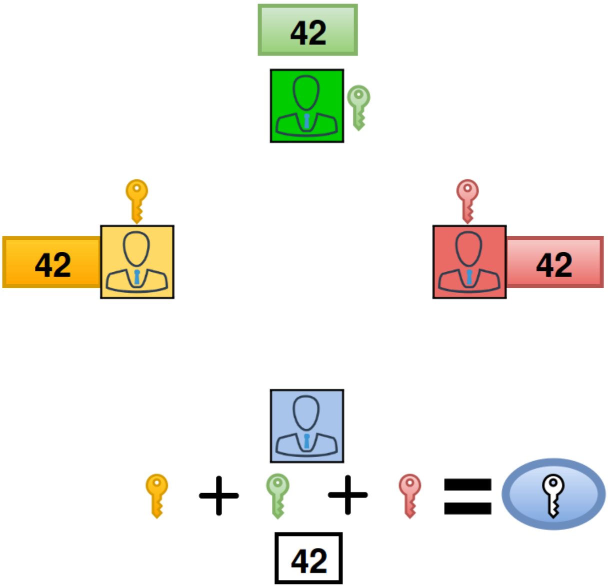

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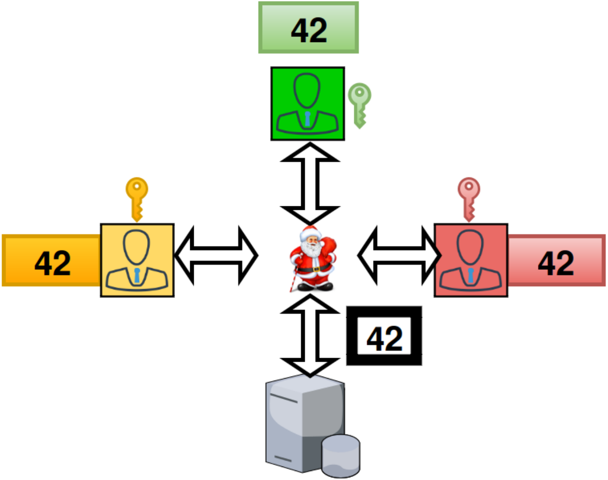


Take home message

Move data **securely** between clients and MPC engines.

Need a PRF mod p

- Enc / Dec in CTR mode use only PRF calls.
- Avoid the n fold database/key blowup by secret share the key and use a PRF mod p in MPC!
- Why mod p? Conversion between binary and arithmetic shares is expensive.

Other use cases for PRF's in MPC

- Secure database joins [LTW13].
- Oblivious RAM [LO13].
- Searchable symmetric encryption, order-revealing encryption [BCO'N11, BLRSZZ15, CLWW16, BBO'N07, CJJKRS13].

What we have done

Benchmark and **create new protocols** using PRF's within SPDZ protocol.

Why SPDZ?

- MPC protocol with active security.
- 200 times faster pre-processing phase [KOS16].
- It is open source!
  `https://github.com/bristolcrypto/SPDZ-2`.

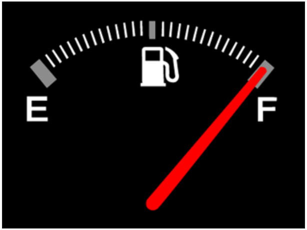
MPC with secret sharing 101

- Each party P_i has $[a] \leftarrow a_i$ s.t. $a = \sum_{i=1}^{n} a_i$.
- Triples generation: $[a] = [b] \cdot [c]$
- Random bits and squares: $[b]$, $[s^2]$.

Preprocessing Phase

MPC with secret sharing 101

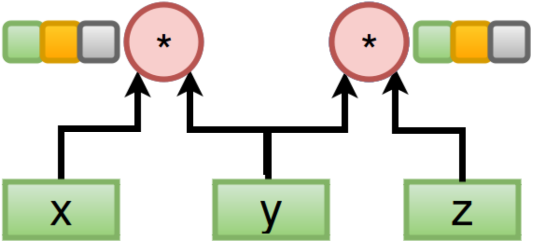
- Use 1 triple for each multiplication gate.
- Number of communcation rounds is given by the multiplicative depth.



Online Phase

Circuit Evaluation in SPDZ

Circuit Evaluation in SPDZ



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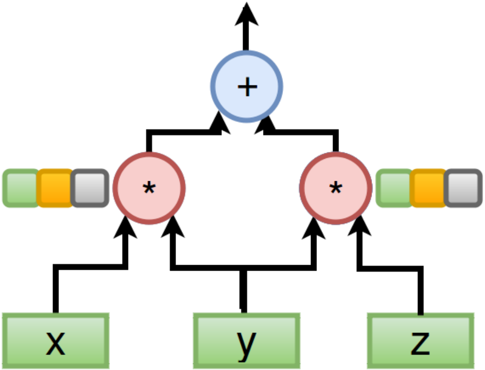

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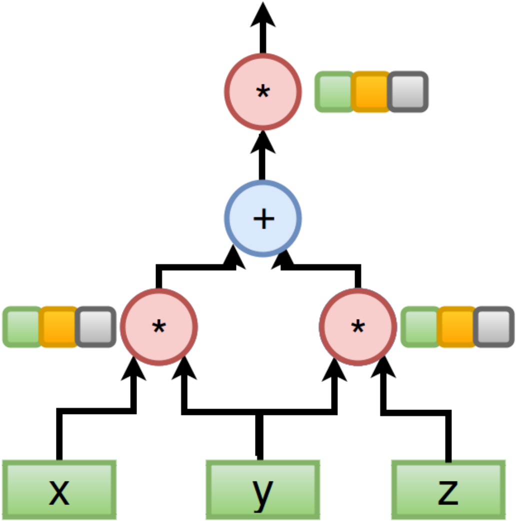

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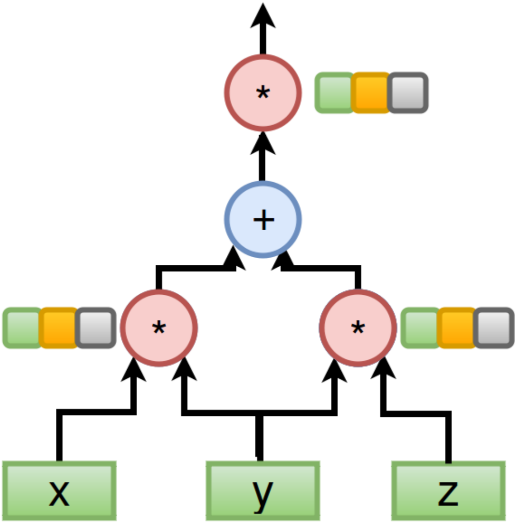

3 triples; 2 rounds.

What PRF's have we looked at?

- AES [DR01].
- LowMC (Low Multiplicative Complexity) [ARS+15].
- Naor-Reingold PRF [NR04].
- MiMC (Minimum Multiplicative Complexity) [AGR+16].
- Legendre PRF [Dam88].

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Let's play a game

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AES - de-facto benchmark

- 960 multiplications
- 50 rounds
- Operations done in $\mathbb{F}_{2^{40}}$.

PRF on blocks

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PRF on blocks

5 blocks/s

AES - de-facto benchmark

- 960 multiplications
- 50 rounds
- Operations done in $\mathbb{F}_{2^{40}}$.

PRF on blocks

8ms latency

AES - de-facto benchmark

- 960 multiplications
- 50 rounds
- Operations done in $\mathbb{F}_{2^{40}}$.

PRF on blocks

530 blocks/s throughput

AES - de-facto benchmark

- Compare the PRF's mod p with AES only for benchmarking purposes.
- In real world we want to keep all data in $\mathbb{F}_p$.

Naor-Reingold PRF

$$F_{\mathsf{NR}(n)}(\mathbf{k}, \mathbf{x}) = g^{k_0 \cdot \prod_{i=1}^{n} k_i^{x_i}}$$

where $\mathbf{k} = (k_0, \ldots, k_n) \in \mathbb{F}_p^{n+1}$ is the key.

Naor-Reingold PRF

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where $\mathbf{k} = (k_0, \ldots, k_n) \in \mathbb{F}_p^{n+1}$ is the key.

Fortunately, in some applications the output must be public!

Naor-Reingold PRF

- Active security version for public output.
- Why EC? Smaller modulus.
- $2 \cdot n$ multiplications.
- $3 + \log n + 1$ rounds.

EC based PRF

Naor-Reingold PRF

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- 7 rounds [BB89, CH10].

EC based PRF in constant round

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5 evals/s

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EC based PRF in constant round

4.3ms latency

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EC based PRF in constant round

370 blocks/s throughput

Naor-Reingold PRF

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- Why EC? Smaller modulus.
- $4n + 2$ multiplications.
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EC based PRF in constant round

Results have shown that over 70% of the time was spent on EC computations.
Computation is the bottleneck, not communication!

MiMC - How does it work?

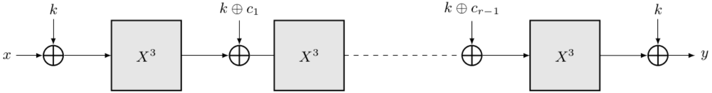

Fig. 1: r rounds of MiMC-n/n

[AGR+16]

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worlds

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MiMC PRF - works in both worlds

34 blocks/s

MiMC PRF

- 146 multiplications
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MiMC PRF - works in both worlds

6ms latency

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worlds

9000 blocks/s throughput - **16x** AES

Legendre PRF

In 1988, Damgård conjectured that this sequence is pseuodarandom starting from a random seed k.

$$\left(\frac{k}{p}\right), \left(\frac{k+1}{p}\right), \left(\frac{k+2}{p}\right), \ldots$$

Legendre PRF - 1 bit output

- $\log p$ multiplications.
- $\log p$ rounds.

Legendre PRF - old version

Legendre PRF - 1 bit output

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

Legendre PRF - new version

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Legendre PRF - new version

1225 evals/s - **250x** AES

Legendre PRF - 1 bit output

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

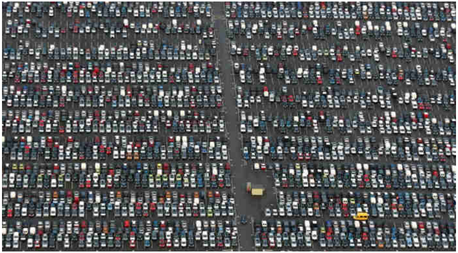
Legendre PRF - new version

0.3ms latency - **25x** faster AES

Legendre PRF - 1 bit output

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

Legendre PRF - new version



202969 blocks/s throughput - **380x** AES

How does it work?

Protocol Π_{Legendre}

Let α be a fixed, quadratic non-residue modulo p, i.e. $\left(\frac{\alpha}{p}\right) = -1$.

Eval: To evaluate $F_{\text{Leg(bit)}}$ on input $[x]$ with key $[k]$:

1. Take a random square $[s^2]$ and a random bit $[b]$
2. $[t] \leftarrow [s^2] \cdot ([b] + \alpha \cdot (1 - [b]))$
3. $u \leftarrow \text{Open}([t] \cdot ([k] + [x]))$
4. Output $[y] \leftarrow \left(\frac{u}{p}\right) \cdot (2[b] - 1)$

Securely computing the $F_{\text{Leg(bit)}}$ PRF with shared output

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Securely computing the $F_{\text{Leg(bit)}}$ PRF with shared output

Security of Legendre PRF

Is it secure?

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Is it secure?

Yes, we give a reduction to the SLS problem: Given $\left(\frac{k+x}{p}\right)$, find x.

Summary

- We have **efficiently** solved the problem of sending data between MPC engines.
- PRF's mod p in MPC are fast! Can you find other applications built on top of these?
- For proofs, WAN timings, other details, check out our paper!

Thank you!